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END-POINT BIAS IN TREND-CYCLE DECOMPOSITIONS: AN APPLICATION TO THE REAL EXCHANGE RATES OF TURKEY

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ABSTRACT Estimating a robust and stable trend is an important challenge for policy analysis. In this paper, we compare alternative approaches by estimating the cyclical component of the real exchange rate series of Turkey. Comparison criterion is the sensitivity of the estimated cycle to additional data points. A formal test reveals that cycle values obtained with all methods change substantially upon new data arrivals. To rank the performance of alternative methods, additional measures underlining the co-movement of real-time cycles with the cyclical values when additional data are available, and the magnitude of end-point bias are developed. These criteria show that an unobserved components approach, which assumes orthogonal trend and cycle innovations and a fixed share of trend shocks on the real appreciation fluctuations of 10%, dominates alternative filtering methods.
JEL C22; E37; F31
Keywords Trend-cycle decompositions, Real exchange rates, Stochastic trend

ÖZ İktisat politikası analizi açısından, zaman serisi eğilimlerinin tutarlı ve kararlı tahmini güçlük arz etmektedir. Bu ampirik çalışmada Türkiye reel döviz kuru serisinin çevrimsel bileşen tahminlerini elde etmede farklı yaklaşımların karşılaştırılması yapılmaktadır. Karşılaştırma kriteri olarak, tahmin edilen çevrimsel bileşenin seriye eklenen verilere duyarlılığı belirlenmiştir. Yapılan testler sonucu, elde edilen çevrimsel değerlerin, bütün yöntemler için seriye eklenen yeni verilere duyarlı olduğu bulunmaktadır. Yöntemleri etkinlikleri açısından sıralamak için, gerçekleşen serilerin çevrimsel bileşen tahminleri ile seriye yeni veriler eklendiğinde elde edilen çevrimsel bileşen tahminlerinin birbiriyle olan bağıntısı hesaplanmış ve bitiş-noktası yanlılık değerleri karşılaştırılmıştır. Bu kriterlere göre, eğilim ve çevrim şoklarının birbirine dik olduğu ve reel döviz kuru değişimi oynaklığının %10'unun eğilim şoklarından kaynaklandığı varsayımları altında, gözlenmeyen bileşenler yaklaşımının alternatif filtreleme yaklaşımlarından üstün olduğu bulunmaktadır.
EĞİLİM-DÖNGÜ AYRIŞTIRMALARINDA BİTİŞ NOKTASI YANLILIĞI: TÜRKİYE REEL DÖVİZ KURUNA UYGULAMA
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Anahtar Kelimeler Eğilim-çevrim ayrıştırmaları, Reel döviz kuru, Raslantısal eğilim

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1. Introduction

Decomposing economic time series into trend and cyclical components has been among the top priorities of policymakers. For instance, the main pillar of the inflation targeting regime is to bring a discipline to the determination of short-term nominal interest rates in response to deviations of inflation and output from a *target* or *fundamental* level. On the other hand, following the recent global financial crisis, additional economic variables such as the real exchange rate and the ratio of credit to GDP have gained more attention from the policymakers in terms of their cyclical behavior around a *fundamental* level.[1]

In order to flesh out the real-time macroeconomic data and extract cyclical components for policy analysis, several methods have been developed by economists. However, many trend-cycle decompositions suffer from an end-point problem, which might be characterized by the exaggerated impact of the terminal data point on the obtained trend. Specifically, if the purpose of the analysis is to document and study the properties of the cyclical component, one can simply omit the terminal data points of the series. On the other hand, if the trend is used for policy purposes, then the terminal data point is likely to be the one, which is particularly interesting.

Choosing the appropriate filtering method is especially important as information on the future path of the economy is missing, when the policy decisions are made. Accordingly, it is only when new data in future periods become available that the trend-cycle decomposition becomes more robust and gets stabilized. The value of the trend in the terminal period might change significantly when new data become available, irrespective of whether new data points are driven by cyclical or structural factors.

The goal of this study is to compare the performance of selected filtering methods in terms of the degree of the end-point problem that they display.[2] We carry out our experiments in a specific application of calculating the cyclical component of the real exchange rates for Turkey. Consequently, this exercise also relates to the permanent equilibrium exchange rate (PEER) literature. In this literature, the trend of the real exchange rates series is

[1] See Drehmann et al. (2010), Aikman et al. (2012), Başçı (2013) and IMF World Economic Outlook (2013).
[2] There is also a very important discussion on the sensitivity of business cycle facts to the filtering methods used. Canova (1994) shows the sensitivity of US business cycle turning points to the filtering methods used. Canova (1998) reports that stylized facts about the US economy vary widely across different detrending methods. In this paper, we focus on the sensitivity of cycle estimates to the new data arrivals.

defined to be a long-run equilibrium value and the cyclical component is referred as deviations from equilibrium real exchange rate or real exchange rate misalignments.[3] In this study, we compare the performance of different methods in order to extract the cyclical component from the real exchange rate series of Turkey.

The paper is organized as follows. Next section describes the data used, Section 3 explains the filtering methods applied. In Section 4, we define different measures of comparison for these methods and discuss the results. Section 5 concludes the paper.

2. Data

We use monthly real exchange rate series of Turkey. Real effective exchange rate index of the Turkish lira vis-à-vis all trading partners of Turkey (RER^{all}) is the series used in the benchmark analysis. The second series used in the study is the real effective exchange rate index of the Turkish lira vis-à-vis trading partners of Turkey, that are classified as advanced economies (RER^{adv}).[4] These indices are calculated and published by Central Bank of Turkey. The sample period starts in January 2003 and ends by December 2012 for both series.

3. Trend-Cycle Decompositions

In this section, we describe the filtering methods used to extract the cyclical component of real exchange rates. A simple method is to run the following regression,

$$y_t = c + \gamma t + \epsilon_t$$

where y_t is the natural logarithm of the real exchange rate and γ gives an average appreciation rate observed in the real exchange rate series of Turkey. We name the trend value $(c + \gamma t)$ calculated by this method as the OLS (Ordinary Least Squares) trend and treat innovations as the cyclical component. Next, we briefly describe the methods in which we assume a stochastic trend, that is, the Beveridge-Nelson Decomposition and the Unobserved Components Approach, and then we proceed with explaining more conventional filtering methods: such as the Hodrick-Presscott Filter and the Wavelet Analysis.[5]

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[3] Huizinga (1987) has been the first study to interpret the cyclical component values extracted by a Beveridge-Nelson decomposition as the real exchange rate misalignments. See MacDonald (2000) and Bussiere et al. (2010) for a detailed discussion on the equilibrium exchange rate estimation literature.

[4] IMF classification at 2009 is used. Advanced economies are Austria, Belgium, Czech Republic, Denmark, Finland, France, Germany, Greece, Ireland, Israel, Italy, Japan, Netherlands, Portugal, Slovakia, Spain, South Korea, Sweden, Switzerland, Taiwan, UK, and US. See Saygılı et al. (2010) for a detailed description of calculations.

[5] See Ekinci et al. (2013) for a detailed description of methods.

3.1. Beveridge-Nelson (BN) Decomposition

This approach assumes that real exchange rates are expressed as the sum of two components,

$$y_t = \tau_t + c_t,$$

where τ_t represents the long-run trend, which follows a random walk as in Beveridge and Nelson (1981) and c_t denotes the cyclical component. The trend component in BN decomposition is defined as the infinite horizon expectation of the time series, that is $\tau_t \equiv \lim_{j\to\infty} E_t\left[y_{t+j}\right]$.[6]

3.2. Unobserved Components (UC) Approach

UC method treats the trend and cycle components as unobservable variables in the state space. In this paper, we make the following assumptions in order to extract the cycle: (i) the cyclical component is assumed to follow an AR(1) process and (ii) trend and cycle innovations are assumed to be uncorrelated.[7] Next, to estimate the model, we fix the weight of the trend component in the real exchange rate appreciation fluctuations at 10 percent.[8]

3.3. Hodrick-Prescott (HP) Filter

HP filter is a widely-used tool in modern macroeconomics to obtain a smoothed non-linear representation of a time series. HP filter treats the observed time series as the sum of a long term growth component and a cyclical component. To extract an estimate for the cycle, HP filter minimizes the following penalty function,

$$\min_{[\tau_t]_{t=1}^{t=T}} \sum_{t=1}^{T}(y_t - \tau_t)^2 + \lambda \sum_{t=2}^{T}[(\tau_t - \tau_{t-1})^2 + (\tau_{t-1} - \tau_{t-2})^2],$$

where λ is a smoothing parameter. When $\lambda = 0$, the trend value is equal to the time series, i.e., $\tau_t = y_t$. As λ gets larger, trend gets closer to a linear OLS estimate. Customary values of λ in the literature are; 100 for annual data, 1600 for quarterly data, and 14400 for monthly data. Since we use monthly data, we set λ=14400 in the analysis.

3.4. Wavelet Analysis

Multi-resolution wavelet analysis is a useful tool for studying the time and frequency properties of an economic time series.[9] Using a wavelet filter, a time series y_t can be decomposed as

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[6] We calculate the trend by implementing the methods described in Morley (2002).

[7] Similar set of assumptions are made for identification purposes in Clark (1987), Harvey (1985), and Watson (1986).

[8] Ekinci et al. (2013) shows that fixing the trend share of real appreciation volatility at 10% gives a similar volatility of cyclical component relative to the other filtering methods.

[9] Following Akkoyun et al. (2012), we choose Symlet-8 family in the filtering procedure, which produces economically interpretable cycles and optimizes smoothness and symmetry.

$$y_t = \bar{y}_t + \sum_{j=1}^{N} c_t^j,$$

where $\bar{y}_t$ captures the cyclical movements longer than 2^{N+1} periods, and c_t^j represents the cycles that last between 2^j and 2^{j+1} periods. In our analysis, we set N=5. Therefore, we define the trend component, $\tau = \bar{y}_t$, as fluctuations that last longer than 64 months and lump the short term fluctuations into the cyclical component, $c_t = \sum_{j=1}^{5} c_t^j$.

4. Results

To examine the degree of the end-point bias for selected filtering methods, we conduct a test proposed by Mohr (2005). Specifically, we run a regression where real-time cycle values are independent variables and cycle values with additional periods are dependent variables, that is,

$$c_{t,t} = \alpha + \beta c_{t,t+k} + \epsilon_t,$$

where second subindex represents the last period of the sample used to obtain the cyclical component. End-point bias is defined as the difference between the real-time cycle and the cyclical value for the extended sample,

$$e_{t,k} \equiv c_{t,t} - c_{t,t+k}.$$

We use the cycle values after January 2008 in running the Mohr (2005) test where the null hypothesis is $H_0 : \alpha = 0$ and $\beta = 1$. The test indicates how the real-time cyclical components are related to the estimates obtained with additional data. Mohr (2005) argues that cycle values obtained by using the full sample are the *true* values for the cyclical component, and therefore utilizes full sample to calculate the dependent variable. From a practical point of view, adding too many observations would lead to a test of a *mid-point bias* instead of an end-point bias.[10] We report our results by including 3, 6, and 12 additional observations in our tests. The results are reported in Tables 1 and 2 for the samples RER^{all} and RER^{adv}, respectively. We observe that the null hypothesis is rejected for all methods and alternative horizons with the exception of 6-months horizon for the HP filter and wavelet analysis methods. Consequently, this test indicates the presence of an end-point bias for all methods. However, test values do not allow us to rank the reliability of methods. Therefore, we proceed to looking at some additional measures in order to compare these alternative approaches in extracting a robust cyclical component.

[10] Our results remain unchanged when the cyclical values with full sample are used.

Table 1. End-Point Bias Test for RER^{all} Sample

Method		3 months		6 months		12 months	
BN		α	β	α	β	α	β
		-0.98	0.95	-1.84	0.89	-3.33	0.78
		(0.19)	(0.03)	(0.24)	(0.03)	(0.21)	(0.03)
	F-statistic	16.32		33.8		130.12	
HP		α	β	α	β	α	β
		-0.26	1.22	-0.80	1.04	-1.76	0.74
		(0.21)	(0.05)	(0.40)	(0.10)	(0.49)	(0.04)
	F-statistic	12.31		2.28		10.17	
OLS		α	β	α	β	α	β
		-0.44	1.07	-1.29	1.08	-3.38	0.96
		(0.10)	(0.01)	(0.20)	(0.03)	(0.28)	(0.04)
	F-statistic	85.43		63.95		83.35	
UC		α	β	α	B	α	β
		-0.38	0.98	-0.68	0.98	-1.45	0.92
		(0.06)	(0.01)	(0.08)	(0.01)	(0.10)	(0.02)
	F-statistic	32.04		59.05		135.37	
Wavelet		α	β	α	β	α	β
		-0.23	1.15	-0.76	0.89	-1.61	0.64
		(0.21)	(0.06)	(0.39)	(0.09)	(0.44)	(0.07)
	F-statistic	5.04		2.34		18.81	

Notes: RER^{all} is the trade weighted real exchange rate index for Turkey against all trading partners, calculated by Central Bank of Turkey. We regress the real-time cycle to the cycles with k additional periods, $c_{t,t} = \alpha + \beta c_{t,t+k} + \epsilon_t$. Standard errors of the regression coefficients are reported in parentheses. Null hypothesis for F-test is H_0 : α =0 and β =1.

Table 2. End-Point Bias Test for RER^{adv} Sample

Method		3 months		6 months		12 months	
BN		α	β	α	β	α	β
		-0.88	0.96	-1.69	0.90	-3.19	0.78
		(0.19)	(0.03)	(0.24)	(0.03)	(0.21)	(0.03)
	F-statistic	13.78		27.34		113.30	
HP		α	β	α	β	α	β
		-0.27	1.22	-0.84	1.03	-1.84	0.74
		(0.21)	(0.05)	(0.42)	(0.10)	(0.50)	(0.09)
	F-statistic	12.83		2.30		10.75	
OLS		α	β	α	β	α	β
		-0.42	1.08	-1.24	1.08	-3.28	0.97
		(0.10)	(0.01)	(0.20)	(0.03)	(0.28)	(0.04)
	F-statistic	81.75		57.27		74.24	
UC		α	β	α	β	α	β
		-0.28	1.00	-0.54	0.99	-1.22	0.94
		(0.05)	(0.01)	(0.08)	(0.01)	(0.09)	(0.02)
	F-statistic	27.24		44.50		97.12	
Wavelet		α	β	α	β	α	β
		-0.10	1.16	-0.45	0.90	-1.17	0.65
		(0.22)	(0.05)	(0.40)	(0.09)	(0.45)	(0.07)
	F-statistic	4.87		1.07		14.59	

Notes: RER^{adv} is the trade weighted real exchange rate index for Turkey against advanced economies, calculated by Central Bank of Turkey. We regress the real-time cycle to the cycles with k additional periods, $c_{t,t} = \alpha + \beta c_{t,t+k} + \epsilon_t$. Standard errors of the regression coefficients are reported in parentheses. Null hypothesis for F-test is H_0 : α =0 and β =1.

One important measure to rank alternative methods is to look at the relationship between real-time cycle estimates and the cyclical components obtained with additional data. To this end, correlations between real-time cycles and estimates with 3, 6, and 12 additional observations after January 2008 are reported in Table 3. We observe a decline in the co-movement of real-time and extended sample estimates as the number of additional periods increases. The table indicates that HP filtering and wavelet analysis show relatively poor performance. Especially when horizons get longer, lower correlations between real-time cycles and cyclical values with additional observations emerge. On the other hand, stochastic trend methods, such as the BN decomposition and the UC approach, and the OLS method display better performance as they deliver larger correlations for all horizons.

Table 3. Correlations of Real-Time Cycles and Cyclical Component with Additional Data

	RER^{all}			RER^{adv}		
	3 months	6 months	12 months	3 months	6 months	12 months
BN	0.98	0.97	0.97	0.98	0.96	0.97
HP	0.96	0.85	0.78	0.96	0.84	0.78
OLS	1.00	0.98	0.96	1.00	0.98	0.99
UC	1.00	1.00	0.99	1.00	1.00	0.99
Wavelet	0.95	0.82	0.79	0.95	0.82	0.80

Notes: RER^{all} is the trade weighted real exchange rate index for Turkey. RER^{adv} is the real exchange rate index for Turkey against the advanced economies. Both series are calculated by Central Bank of Turkey and cover the period between January 2003 and December 2012. Real-time cycles and cyclical values with additional data are obtained for the periods between January 2008 and December 2011.

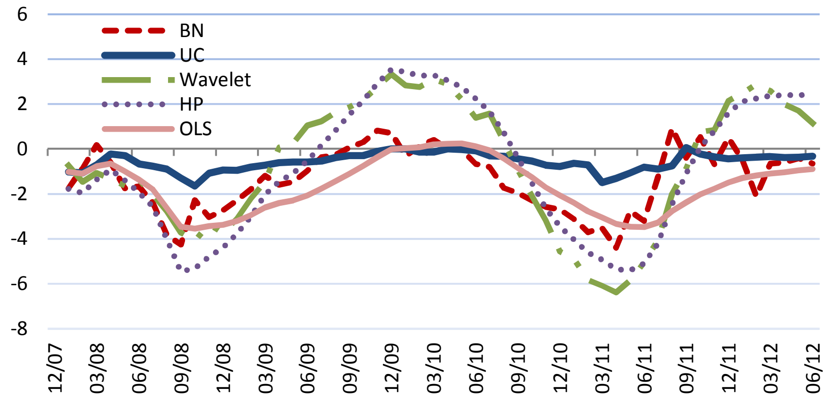
Another measure for comparison is the magnitude of ex-post errors. We report the standard deviation of end point bias, $e_{t,k}$, for each method with different horizons in Table 4. According to this metric, UC approach performs better than BN decomposition for all horizons. OLS method, on the other hand, performs better than the UC approach, but only at a 3 months horizon. For an illustration, we plot the end point bias produced by all methods within a horizon of 6 months (k=6) in Figure 1. It is clear that the UC approach dominates the alternative filtering methods, since it produces a much smaller end-point bias.

Table 4. Volatility of End-Point Bias with Alternative Methods

RER^{all}			
	3 months	6 months	12 months
BN	1.37	1.39	1.69
HP	1.92	2.90	3.52
OLS	0.66	1.17	1.76
UC	1.17	0.40	0.51
Wavelet	1.69	2.85	3.55
RER^{adv}			
	3 months	6 months	12 months
BN	1.40	1.44	1.75
HP	1.96	2.99	3.62
OLS	0.68	1.22	1.81
UC	1.07	0.39	0.50
Wavelet	1.75	2.94	3.64

Notes: We report the standard deviations of end-point bias values for alternative methods with different horizons. RER^{all} is the trade weighted real exchange rate index for Turkey. RER^{adv} is the real exchange rate index for Turkey against the advanced economies. Both series are calculated by Central Bank of Turkey and cover the period between January 2003 and December 2012. Real-time cycles and cyclical values with additional data are obtained for the periods between January 2008 and December 2011.

Figure 1. End-Point Bias at 6-months Horizon for Alternative Filtering Methods (RER^{all})



Estimated trend values with the BN decomposition, the UC approach, and the OLS method are plotted with the actual real exchange rate series (with respect to all trading partners of Turkey) in Figure 2. We observe a long-term appreciation in the real exchange rate possibly due to the Balassa-Samuelson effect (an increase in the price of non-tradable goods due to the catch-up in the relative income) and a higher degree of quality bias in inflation compared to advanced economies.[11] Finally, in order to quantify

[11] See Arslan and Ceritoğlu (2011) for the impact of quality bias in the calculation of inflation.

this prior, we also report the *estimated average real appreciation* rates of Turkish lira implied by the BN decomposition, the UC approach, and the OLS method in Table 5. For the analysts who examine the real exchange rate dynamics of Turkey, these real appreciation rates are crucial. In particular, a trend value for RER^{all} index estimated with the UC approach should be expected to be 2.36% higher in 12 months.

Figure 2. Trend Values for RER^{all} Series with BN, UC and OLS Methods

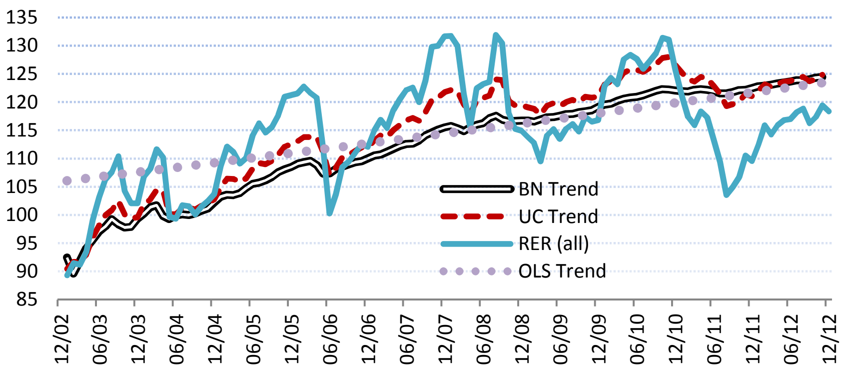


Table 5. Real Appreciation Rate for Turkey

	RER^{all}	
Method	monthly	annual
BN	0.16	2.00
OLS	0.12	1.55
UC	0.19	2.36
	RER^{adv}	
Method	monthly	annual
BN	0.25	3.00
OLS	0.22	2.67
UC	0.27	3.30

Notes: RER^{all} is the trade weighted real exchange rate index for Turkey. RER^{adv} is the real exchange rate index for Turkey against the advanced economies. Both series are calculated by Central Bank of the Republic of Turkey and cover the period between January 2003 and December 2012. Real-time cycles and cyclical values with additional data are obtained for the periods between January 2008 and December 2011.

5. Conclusion

Trend-cycle decompositions are essential tools for policy analysis targeting macroeconomic and/or financial stability. To examine and compare the reliability of different filtering methods, we apply a number of techniques to the real exchange rate series of Turkey. A formal test is conducted by using real-time cycles and the cyclical values obtained with additional data. Test results indicate that all methods display a serious degree of end-point bias, i.e., the estimated cyclical values are sensitive to the terminal data points. To rank the methods, we examine the co-movement of the real-time cycles and the estimates obtained by using additional data within horizons of 3, 6, and 12 months, respectively. We also compare the methods along the dimension of the ex-post magnitude of the end-point bias. Results show that a decomposition based on the unobserved components approach, which assumes (i) trend and cycle innovations are orthogonal and (ii) the share of trend shocks on the real depreciation rate fluctuations is fixed at 10%, dominates the other filtering methods considered in this study. Finally, the average real appreciation rate is estimated to be around 2% for Turkish lira in the trade-weighted general real exchange rate index, and around 3% for the real exchange rate vis-à-vis advanced countries.

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