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# Predictability of Turkish Foreign Exchange and its Implications to Option Pricing and Arbitrage Opportunities

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**Abstract**

Hurst (1951) developed Rescaled range analysis to determine long-memory effects and fractal Brownian motion in time series. Rescaled range (R/S) analysis measures how the distance covered by a particle increases as we look at longer and longer time scales. For Brownian motion the distance increases by the square root of time. An increase with any other ratio asserts a non-random effect. To establish the option pricing formula for a non-dividend paying stock, Black&Scholes (1973) made the assumption that the underlying stock price follows a log-normal distribution with one dimensional Brownian motion. This implies that the ratio of increase has to be by the square root of time for randomness. This paper investigates the efficiency of Turkish Foreign Exchange by using Rescaled Range analysis and its further implications to option pricing. Rescaled Range analysis is applied to daily observations of the US dollar and Euro against Turkish lira and US dollar-Euro parity. The existence of arbitrage opportunities is investigated.

*JEL Classification:* C13, C14, G12.
*Keywords:* Turkish Foreign Exchange, Rescaled Range Analysis, Hurst exponent, Brownian motion, Currency Options.

## 1. Introduction

A lot of research has been undertaken on the long-term dependence of stock returns and volatility as well as the term structure of volatility in recent years. This is the main building block of most statistical prediction techniques since they make the assumption of independent and identically distributed data. Unfortunately, classical techniques are not well suited to identify any nonlinear structure in the data. Financial market data exhibits leptokurtic behaviour and this violates with the assumption of being independent and identically distributed.

In many ways, nonparametric statistical techniques provide an alternative to test such nonlinear structure since they do not impose assumptions. One such nonparametric technique is rescaled range analysis first introduced by Hurst (1951) in order to investigate how reservoir capacity fluctuates over time. Subsequently rescaled range analysis is refined to handle economic time series by Mandelbrot (1972, 1982), Mandelbrot and Wallis (1969) and Lo (1991). Also, Peters (1994) applied Rescaled Range analysis to Dow Jones industrial index over the period 1988 and 1992.

The purpose of this paper is to search anomalies such as the absence of the random walk pattern and the lack of independently and identically distributed asset price changes in the Turkish Foreign Exchange. Presence of long-term dependence in the US dollar and Euro against Turkish lira and US dollar-Euro parity is investigated. Deviations from Brownian motion and implications to option pricing and arbitrage opportunities are asked.

## 2. Rescaled Range Analysis and Hurst Exponent

Let X(t) be the price of a stock on time t and r(t) be the logarithmic return calculated as follows.

$$r(t) = \log\left(\frac{X(t+1)}{X(t)}\right) \tag{1}$$

Asset returns are denoted as $\{r(1), r(2), \ldots, r(\tau)\}$ and $\bar{r}_\tau$ is the sample mean $\bar{r}(\tau) = 1/\tau \sum_\tau r(\tau)$ on the time span considered. To estimate the Hurst exponent, the range of the accumulated deviations from the average level has to be calculated first. This can be obtained by calculating the difference between the maximum and minimum cumulative deviations over $\tau$ periods (Peters 1989, 33).

$$(R/S)_\tau = \frac{1}{S_\tau}\left[\max_{1\le t\le \tau}\sum_{k=1}^{t}\left(r(k)-\bar{r}_\tau\right) - \min_{1\le t\le \tau}\sum_{k=1}^{t}\left(r(k)-\bar{r}_\tau\right)\right] \tag{2}$$

In equation (2), $S_\tau$ denotes the usual standard deviation on the time span considered.

$$S_\tau = \sqrt{\frac{1}{\tau}\sum_t \left(r(t)-\bar{r}_\tau\right)^2} \tag{3}$$

The range depends on the time period considered. It is expected that the value of R increases as $\tau$ increases. R is divided by the standard deviation in order to standardize the observations. Hurst (1951) found that $(R/S)_\tau$ could be estimated by the following empirical law.

$$(R/S)_\tau = (\tau)^H \tag{4}$$

Using a logarithmic transformation, the equation becomes as follows. H can be estimated by performing an ordinary-least-squares (OLS) regression between log($\tau$) and log(R/S).

$$\mathrm{Log}(R/S)_\tau = H\log(\tau) \tag{5}$$

Specifically, if H is between zero and 0.5, anti-persistent behaviour exists which means there is a negative dependence between the increments. If a trend has been positive in the last period, it is more likely to be negative than positive in the next period. But the H value greater than 0.5 indicates that some periods above or below the theoretical mean are extraordinarily long. If the value of H is greater than 0.5, the time series exhibit persistent behaviour. If the trend has been positive in the last observed period, it is likely to be positive in the next period. The level of persistence is measured by how far H is above 0.5.

For very long series, or in other words by increasing $\tau$, H tends to converge to value 0.5. Therefore, the regression referred to above has to be performed on the data prior to convergence of H to 0.5. Also, the correlation between periods can be calculated as follows (Pallikari and Boller 1999, 27):

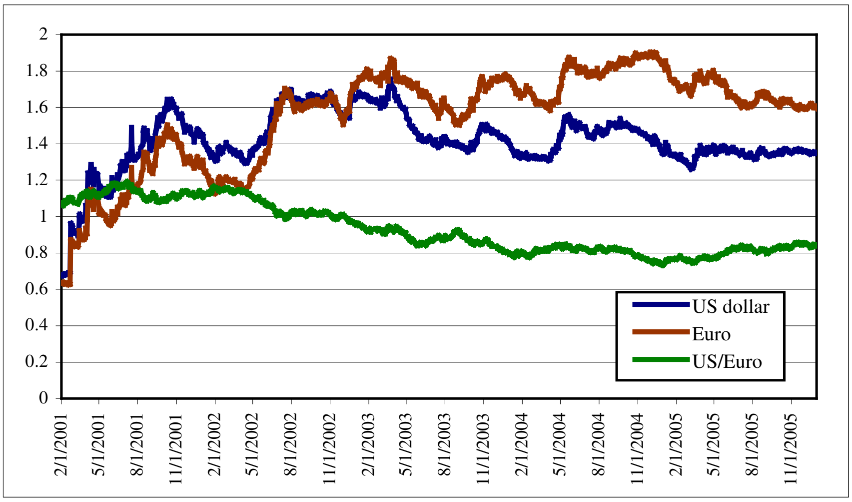
$$C_\tau = 2^{(2H-1)} - 1 \tag{6}$$

As a result of Brownian motion (H=0.5), $C_\tau$ can be calculated as zero. If the time series exhibit persistent behaviour (H>0.5) $C_\tau$ takes positive values and anti-persistent time series has negative correlation coefficient.

## 3. Empirical Results

This paper applies Hurst's R/S (Rescaled Range) analysis to the US dollar and Euro against Turkish lira and US dollar-Euro parity within the same time horizons. 1235 daily observations are taken into consideration between the dates February 02, 2001 and December 30, 2005 provided by Central Bank of the Republic of Turkey (CBRT). Therefore, the R/S analysis is applied to Turkish data when the YTL floats freely. Log returns for each time series are obtained first and R/S analysis is modelled on a spreadsheet by calculating consecutive sub period Log(R/S) values. Number of daily observations taken into consideration for each sub-period can be seen in the appendix. Following figure illustrates the graphic of each series in order to investigate the time-dependence in foreign exchange volatility and analyze whether the volatility tends to rise or subdue during upward and downward trend.

**Fig. 1. Turkish Foreign Exchange between February, 2001-December, 2005**




ARCH effect does not seem to exist in any of the given time series but a more definite conclusion can be drawn by applying ARCH-LM test to the residuals obtained from OLS regression. ARCH-LM test is conducted and results are gathered in Table 6 in the appendix. Focusing on the results obtained from ARCH-LM tests, null hypothesis is accepted due to the probabilities obtained 0.0850, 0.4873 and 0.5661 respectively for US Dollar, Euro and US Dollar-Euro parity respectively. Therefore, ARCH effect does not seem to exist. Specifically, the volatility does not tend to rise or subdue during the upward and downward trend.

**Table 1**
**Regression Results for Turkish Foreign Exchange**

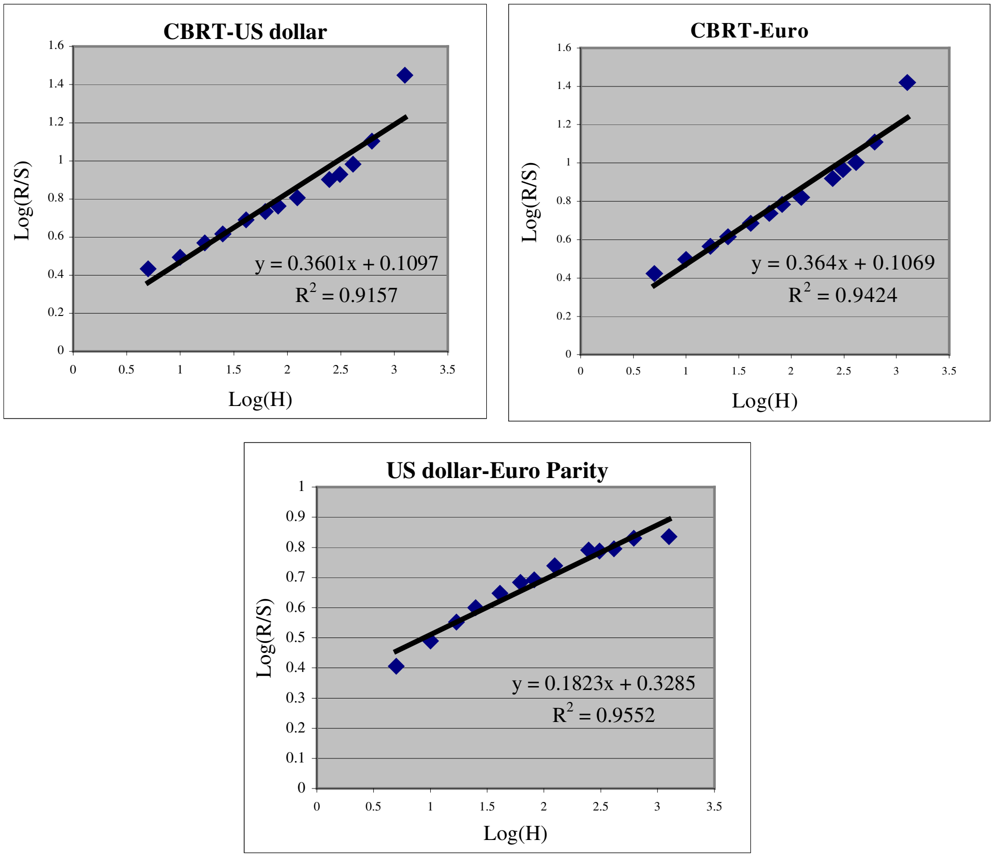
| Parameters | CBRT US dollar | CBRT Euro | US Dollar-Euro Parity |
|---|---|---|---|
| R-squared | 0.915707 | 0.942353 | 0.955219 |
| X coefficient(H) | 0.360076 | 0.364005 | 0.182253 |
| Std. Err. Of Coef. | 0.032939 | 0.027145 | 0.011898 |
| t-stat | 10.93149 | 13.40960 | 15.31791 |
| p-value | 3.02E-07 | 3.69E-08 | 9.14E-09 |
| $C_\tau$ | -0.176322 | -0.171823 | -0.356279 |

Table 1 summarizes the regression results for each time series. H is estimated to be 0.360076, 0.364005 and 0.182253 for CBRT-US dollar, CBRT-Euro and US dollar-Euro parity respectively. Since the Hurst exponent values are lower than 0.5 for each currency, these series exhibit anti-persistent behaviour means, there is a negative dependence between the increments. It is a common sense that, if a trend has been positive in the last period, it is more likely to be negative than positive in the next period. The mean-reverting effect is said to be stronger for the series US dollar- Euro parity since the deviation from 0.5 is higher.

Also, the correlation coefficient helps to analyse the relation between the periods. For example $C_\tau$ coefficient for CBRT-US dollar series -0.176322 implies that 17.6322 percent of changes in the US dollar currency are influenced by the past deviations. The correlation coefficients given above support these claims. Negative correlations prove the anti-persistent behaviour and US dollar-Euro parity has the strongest correlation between periods within the lowest Hurst coefficient ($0<H<0.5$). The high R squared values and the low standard errors illustrate the goodness of fit.

Figure 2 given below, illustrates and gathers the results of the regression analysis. The solid lines exhibit an unbiased random walk within the H value of 0.5 whereas the actual returns deviate slightly from the regression line. The patterns in each series suggest difficulty in forecasting the Turkish foreign exchange over the long term but not impossible. The H values and the regression equations can be compared with the ones obtained from the Hurst analysis.

**Fig. 2. An Illustration of Rescaled Range Analysis**




## 4. Conclusions and Implications

This paper tests for long-term dependence and deviations from random walk hypothesis in the Turkish foreign exchange and its implications to option pricing. Results from R/S analysis indicate that the Turkish foreign exchange does not seem consistent with random walk hypothesis. The Hurst exponent values lower than 0.5 asserts that, the Turkish foreign exchange is anti-persistent or mean-reverting. Findings are consistent with the ones obtained by Aysoy and Balaban (1996). They analyzed a managed float period whereas this paper conducts the analysis when the YTL floats freely. They used daily observations of the US dollar and German mark against Turkish lira between July 2, 1981 and December 29, 1995 and found that the Turkish foreign exchange is anti-persistent or mean-reverting. This implies that the structure of the Turkish foreign exchange seem stable considering the two different time horizons.

Black&Scholes (1973) made the assumption that the underlying stock price follows a log-normal distribution with one dimensional Brownian motion. Findings of this paper prove that each of the currency returns exhibits fractal Brownian motion which is a generalization of the Brownian motion obtained by removing the last condition. The implications of these findings on pricing currency options have to be taken into consideration. As the stochastic process for a foreign currency is the same as that for a stock paying a dividend yield equal to the foreign risk-free rate, the Black&Scholes option pricing formula for a European call option becomes as follows (Hull 2003, 277):

$$C(S_t, t) = S_t.e^{-r_f(T-t)}N(d_1) - Ke^{-r(T-t)}N(d_2) \quad (7)$$

where

$$d_{1,2} = \left(\ln(S_t / K) + (r - r_f \pm \tfrac{1}{2}\sigma^2)(T-t)\right) \Big/ \left(\sigma\sqrt{T-t}\right)$$

So, it is quite obvious that $(T\text{-}t)^{0.5}$ term in the denominator of $d_{1,2}$ becomes $(T\text{-}t)^{0.360076}$ and the $(T\text{-}t)= [(T\text{-}t)^{0.5}]^2$ term becomes $(T\text{-}t)^{0.720152}$ for US dollar. An illustrative example can be used to investigate the effects of deviation from random walk.

**Table 2**
**The Value of a European Call Option on a Non-dividend Paying Stock**

| Case Parameters | Values |
|---|---|
| **Stock Price** | 100 |
| **Strike Price** | 100 |
| **Volatility** | 0.25 |
| **Interest Rate** | 0.10 |
| **Time to Maturity** | 0.50 |

The theoretical price for the derivative is determined as 9.582235 analytically by using the Black&Scholes formula. But transforming the (T-t) terms, one would calculate the option price as 10.243972. This value should be suggested as the fair price of the option. Both the domestic interest rate (r) and foreign interest rate ($r_f$) are rates for maturity (T-t). Establishing Black&Scholes formula given by (7), it is assumed that exchange rates follow a geometric Brownian motion. This asserts the standard deviation to increase by the square root of time. But the findings in the previous section emphasize that currency options are being undervalued on the US dollar and Euro by using equation (7). Black&Scholes (1973) asserts that the fair price is calculated only by equation (7) but under these circumstances arbitrage opportunities for investors would occur.

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# Appendix

**Table 3**
**R/S Analysis for Daily Observations of CBRT-US Dollar Against Turkish Lira**

| T | Logτ | R/S | LogR/S |
|---|---|---|---|
| **1257** | 3.099335 | 28.12715 | 1.449126 |
| **618** | 2.790988 | 12.69633 | 1.103678 |
| **412** | 2.614897 | 9.621475 | 0.983242 |
| **309** | 2.489958 | 8.488323 | 0.928822 |
| **247** | 2.392697 | 7.981192 | 0.902068 |
| **124** | 2.093422 | 6.395327 | 0.805863 |
| **82** | 1.913814 | 5.794433 | 0.763011 |
| **62** | 1.792392 | 5.431767 | 0.734941 |
| **41** | 1.612784 | 4.901246 | 0.690307 |
| **25** | 1.397940 | 4.131750 | 0.616134 |
| **17** | 1.230449 | 3.716718 | 0.570160 |
| **10** | 1.000000 | 3.115592 | 0.493541 |
| **5** | 0.698970 | 2.708854 | 0.432786 |

**Table 4**
**R/S Analysis for Daily Observations of CBRT-Euro against Turkish Lira**

| τ | Logτ | R/S | LogR/S |
|---|---|---|---|
| **1257** | 3.099335 | 26.31351 | 1.420179 |
| **618** | 2.790988 | 12.82328 | 1.107999 |
| **412** | 2.614897 | 10.02021 | 1.000877 |
| **309** | 2.489958 | 9.216386 | 0.964561 |
| **247** | 2.392697 | 8.288173 | 0.918459 |
| **124** | 2.093422 | 6.598474 | 0.819444 |
| **82** | 1.913814 | 6.063480 | 0.782722 |
| **62** | 1.792392 | 5.447186 | 0.736172 |
| **41** | 1.612784 | 4.845168 | 0.685309 |
| **25** | 1.397940 | 4.115066 | 0.614377 |
| **17** | 1.230449 | 3.676750 | 0.565464 |
| **10** | 1.000000 | 3.135109 | 0.496253 |
| **5** | 0.698970 | 2.654739 | 0.424022 |

**Table 5**
**R/S Analysis for Daily Observations of US Dollar-Euro Parity**

| τ | Logτ | R/S | LogR/S |
|---|---|---|---|
| **1257** | 3.099335 | 6.843573 | 0.835283 |
| **618** | 2.790988 | 6.763402 | 0.830165 |
| **412** | 2.614897 | 6.238490 | 0.795080 |
| **309** | 2.489958 | 6.142946 | 0.788377 |
| **247** | 2.392697 | 6.177095 | 0.790784 |
| **124** | 2.093422 | 5.483813 | 0.739083 |
| **82** | 1.913814 | 4.928540 | 0.692718 |
| **62** | 1.792392 | 4.827741 | 0.683744 |
| **41** | 1.612784 | 4.441887 | 0.647568 |
| **25** | 1.397940 | 3.977069 | 0.599563 |
| **17** | 1.230449 | 3.565636 | 0.552137 |
| **10** | 1.000000 | 3.089708 | 0.489917 |
| **5** | 0.698970 | 2.546319 | 0.405913 |

**Table 6**
**ARCH-LM Tests for Residuals**

**US-Dollar**

| **F-Statistic** | 2.970731 | **Probability** | 0.085034 |
|---|---|---|---|

| **Variable** | **Coefficient** | **Std. Error** | **t-Statistic** | **Prob.** |
|---|---|---|---|---|
| **Constant** | 0.949601 | 0.028474 | 33.35004 | 0.0000 |
| **Residual** | 0.049049 | 0.028458 | 1.723581 | 0.0850 |

**Euro**

| **F-Statistic** | 0.482677 | **Probability** | 0.487343 |
|---|---|---|---|

| **Variable** | **Coefficient** | **Std. Error** | **t-Statistic** | **Prob.** |
|---|---|---|---|---|
| **Constant** | 0.977831 | 0.028513 | 34.29430 | 0.0000 |
| **Residual** | 0.019792 | 0.028488 | 0.694749 | 0.4873 |

**US-Dollar/EURO**

| **F-Statistic** | 0.329513 | **Probability** | 0.566051 |
|---|---|---|---|

| **Variable** | **Coefficient** | **Std. Error** | **t-Statistic** | **Prob.** |
|---|---|---|---|---|
| **Constant** | 0.982608 | 0.028564 | 34.40054 | 0.0000 |
| **Residual** | 0.016378 | 0.028532 | 0.574032 | 0.5661 |